Spike-based Expectation Maximization: Supplementary Material

Bernhard Nessler, Michael Pfeiffer, Wolfgang Maass
Institute for Theoretical Computer Science, Graz University of Technology, A-8010 Graz
{nessler,pfeiffer,maass}@igi.tugraz.at

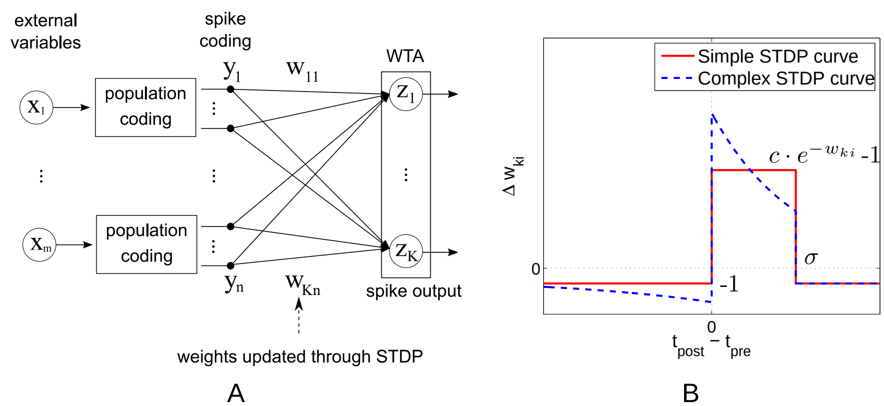

Figure 1: **A)** Architecture for learning with STDP in a WTA-network of spiking neurons. **B)** Learning curve for the two STDP rules that were used (with $\sigma = 10$ms, $c = e^5$). The red curve results from a theoretically optimal approximation of EM within this architecture, whereas the blue-dashed curve represents a variation that is closer to experimental data (see e.g. Fig. 7C for an intermediate pairing frequency in [1]). The synaptic weight w_{ki} is changed in dependence of the difference $t_{post} - t_{pre}$ of the firing times t_{pre} of the presynaptic neuron y_i and t_{post} of the postsynaptic neuron z_k. If z_k fires at time t without a firing of y_i in the interval $[t - \sigma, t + 2\sigma]$, w_{ki} is reduced by 1.

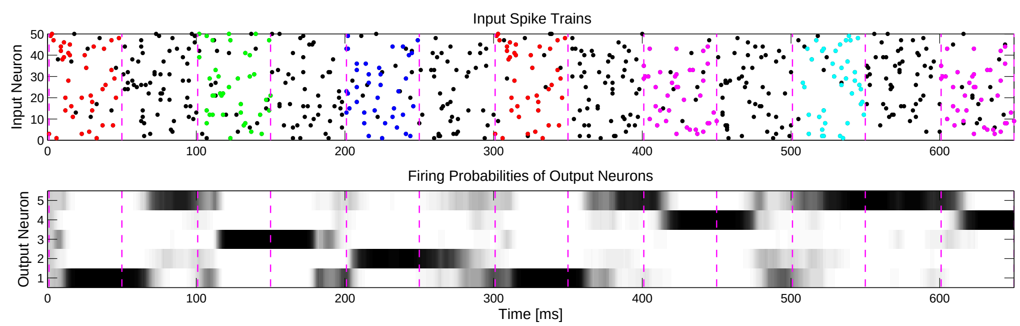

Figure 2: Spike-based EM learns without any supervision to identify 5 fixed spike patterns, embedded in a continuous noisy input stream with constant overall firing rate of 20 Hz. The spike patterns are initially created as 15 Hz Poisson spike trains of 50ms on 50 input neurons, and presented repeatedly in random order, interleaved by 50ms noise intervals. The network learns from a 20 second-long training input to recognize generalizations of the patterns, which are perturbed by 5 Hz random background noise, and to discriminate the embedded fixed patterns from each other The top panel shows 650 ms of a test input, where the spikes of the 5 different patterns are plotted in color, and noise spikes in black. The bottom plot shows that each of the 5 output neurons learns to fire predominantly during one spike pattern (dark pixels illustrate high firing probability).

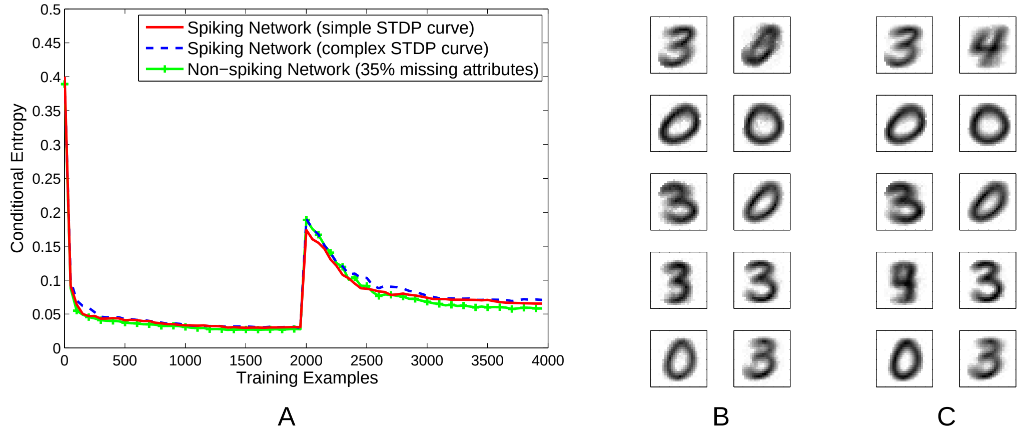

Figure 3: Analysis of the learning progress of spike-based EM for the MNIST dataset of handwritten digits. These results show that the network has learnt without supervision to discriminate handwritten digits, and is able to generalize to new handwritten digits not shown during learning. **A)** Normalized conditional entropy of the correct label conditioned on the network output for two spiking networks with the two variants of STDP learning rules illustrated in Fig. 1B (red solid and blue dashed lines), as well as a non-spiking approximation of the network. According to this analysis the non-spiking network with 35% missing values (green line) is expected to have a very similar learning behavior to the spiking network. First, 2000 random examples of handwritten digits 0 and 3 were presented (for 50ms each) to the network. The next 2000 examples also included random examples of digit 4. **B)** The implicit internal models created by the neurons after 2000 training examples. One can clearly see that neurons created separate internal models for different ways of writing the two digits 0 and 3. **C)** Re-organized internal models after 2000 further training examples that included digit 4. Two output neurons had created internal models for the newly introduced digit 4.

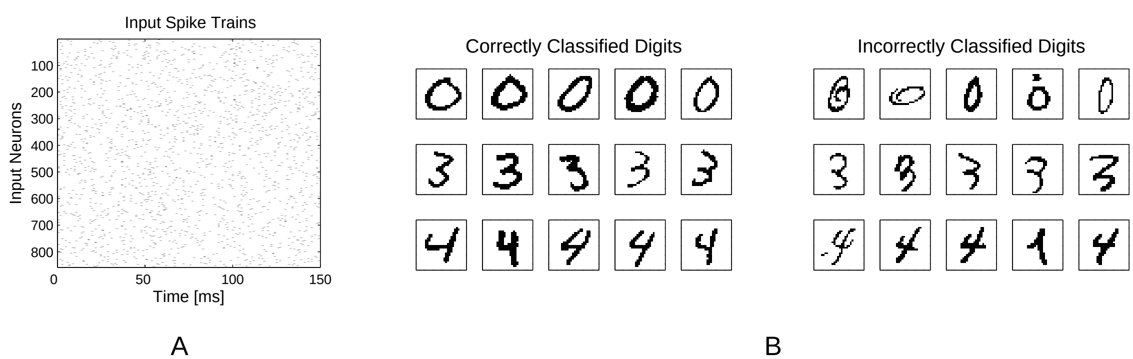

Figure 4: **A)** Example of population encoding of 3 consecutive MNIST digit images by 858 input spike trains, in which every digit is presented for 50 ms. Each binary pixel is encoded by two neurons, spiking with a Poisson rate of 40 Hz for the active state, and 0 Hz otherwise. The setup works similarly well for presentation times of 10 ms. **B)** Examples of input digits for three classes $(0, 3$, and $4)$ that are classified correctly (left) or incorrectly (right). The unsupervised learning achieves a classification error rate of 2.19% on the digits 0 and 3 after 2000 training steps and 3.68% on all three digit classes after 4000 training steps on an independent test set of 10,000 new unseen samples each.

References

[1] P. J. Sjöström, G. G. Turrigiano, and S.B. Nelson. Rate, timing, and cooperativity jointly determine cortical synaptic plasticity. *Neuron*, 32(6):1149–1164, 2001.